Ion Movement During an Action Potential

We know that an action potential (AP) is due to ions crossing the cell membrane in response to changes in permeability. In contrast to the GCSE/A-level view of large numbers 'flooding across' to change the potential by bulk flow of charge, in fact the movement is small – hence even if the Na^+-K^+ pump ceases to function, the existing gradient is sufficient for a large number of APs to be propagated. What follows are a couple of calculations to demonstrate this.

The action of the Na-K ATPase is slightly electrogenic – that is, by pumping three positive charges (Na^+) out for every two (K^+) in, it makes the inside of the membrane negative relative to the outside. For the sake of simplicity we will ignore this effect for the moment[1].

We start with a high Na^+ concentration outside, and high K^+ inside. At the start of the AP the Na permeability (P_{Na}) becomes much higher than the K permeability (P_K). The ratio P_{Na}:P_K is sufficiently high that we can, to a fair approximation, ignore P_K[2].

Initially Na^+ ions will flow down the concentration gradient unopposed. However, as soon as ions start to cross, we are putting charges on opposite sides of a barrier, and the membrane starts to act as a capacitor (a device for storing electrical charges – exactly what we have happening here). Q=CV where Q is the charge stored (in Coulombs), C is the capacitance (in Farads), and V is the potential across the membrane (in Volts). A typical neuronal membrane has a capacitance of about $1\mu F/cm^2$. Assume a typical neuron has a diameter of 10μm, then it will have a circumference of 31μm, giving an area per unit length (1m[3]) of $3.1x10^{-5}m^2$, and so a capacitance per unit length of 0.31μF.

We can use the Nernst equation to work out the membrane potential at equilibrium (i.e. when the potential difference balances the concentration gradient and there is no net flow of ions). This is about +50mV, or about 120mV above the resting potential of -70mV.

We have C and V. The total charge is the charge of each ion, multiplied by the number of ions that move. The charge on a +1 ion is $1.6x10^{-19}$ Coulombs (this is a fundamental constant of nature). If we say that *n* Na ions move, then $n x1.6x10^{-19}$ = CV. This can be rearranged (divide both sides by $1.6x10^{-19}$) to show that $n=2.3x10^{11}$. This looks like a lot as a number, but consider the change in concentration of Na^+ inside the axon it represents. We have a diameter of 10μm, hence a radius of 5μm, and a cross-sectional area of $7.75x10^{-11}m^2$ (πr^2). A unit length will have a volume of $7.75x10^{-11}m^3$, which is $7.75x10^{-8}$litres ($1000cm^3$ in a litre). $2.3x10^{11}$ ions is equal to $3.9x10^{-13}$ moles (divide by Avogadro's constant, $6.02x10^{23}mol^{-1}$). The change in concentration is around 5μM. Hopefully it is reasonably clear that the change in K^+ concentration when the neuron repolarises will be the same. As intracellular K^+

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[1] This isn't entirely unreasonable, especially as other ions and charged particles (especially proteins) are also present.
[2] If we include it, only really need to use Goldman equation instead of Nernst in the later steps. You can try it ($P_{Na}=100P_K$), but it makes a very small change to the figures.
[3] We can take any length, and as long as it is used consistently throughout the calculation the result will be the same. Taking a unit length just makes some of the maths a little easier.

concentration is around 100mM, it is reasonable to say that changes in membrane potential are due to the movement of relatively small numbers of ions.

There is one further point to consider. In calculating the membrane potentials with the Nernst equation, we assumed that the concentrations of ions inside and outside the cell were constant. Does this make the calculation a circular argument? The answer (happily) is no. If the concentrations changed significantly as the ions moved, the concentration gradient would reduce. This would reduce the membrane potential necessary for equilibrium. A smaller potential needs a smaller number of charges to cross the membrane, and so involves a smaller change in concentration than the calculation would suggest – and so assuming the concentrations are effectively constant is reasonable, at least for any one AP.

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