

Logarithms

Introduction

The logarithm ('log') is a mathematical function, fundamentally no more complicated than addition or division.

Terminology

$\text{Log}_a(b)=c$ Log to the base a of b is equal to c

Definition

$\text{Log}_a(b)=c$ means that $a^b = c$

Common types

The bases most commonly used are 10 and e (approx 2.72). Log to base e is often called the natural log of a number, and written as \ln . Log when written without a base specified is usually taken to mean \log_{10} .

Properties

The log of a product is equal to the sum of the logs of the two components.

i.e. $\log(ab) = \log(a) + \log(b)$

The log of an exponential is the log of the base multiplied by the exponent

i.e. $\log(a^b) = b \log(a)$

Uses

Logs are useful to make a linear plot of an exponential function. To take a simple case, consider $y = 20^x$. If we take logs of both sides, we get $\log(y) = x\log(20)$. $\log(20)$ is a constant, so if we plot $\log(y)$ against x we will get a straight line. If we didn't know the value of the constant, the gradient of the line would give it.

Logs can also be used to compress a scale. $\log(10) = 1$, but $\log(10000) = 4$. If plotting data over a wide range of values, the use of logs can effectively compress a scale, making it easier to view all the data on a single graph.

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